

## DEVELOPING STUDENTS' MATHEMATICAL UNDERSTANDING USING GEOGEBRA SOFTWARE

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### Abstract

*Technology is an essential reference in all aspects. Technology provides significant changes in education part, especially in learning mathematics. Technology has an impact on the innovation space in mathematics learning, the problems so far tend to be less attractive, the environment is boring and very tied to reading materials, which results in students still not being able to develop their mathematical thinking process optimally, namely students' mathematical understanding is still low. Students' mathematical understanding is one of the most important capabilities that students should have to solve various problems, both mathematical problems for instance problems related to concepts, operations, principles, and in daily life. So that innovation in learning with the influence of learning technology is important to apply. The aim of this study was to develop students' conceptual understanding of transformation geometry using Geogebra software. This study used an exploratory mixed methods designs that aimed to describe students' understanding of mathematics by learning using Geogebra software. The results of the study indicate that usage Geogebra software can develop students' understanding of mathematical concepts in the transformation geometry, and assisted teachers create an interesting and fun learning process, teachers find the right procedures in using applications such as the use of different software and techniques in learning mathematics, making it easier for students to understand the abstract material of transformation geometry. The results of this study are focused on test analysis, namely exploring students' mathematical understanding, data can also be obtained from interview techniques that explore test answers, in the form of 2 problems related to transformation geometry. The results of the study indicate that are 5 criteria that have met the students' understanding of mathematics from 7 indicators.*

**Keyword:** *Developing, Mathematical Understanding, Geogebra Software.*

### INTRODUCTION

There is no doubt that students should learn mathematics with understanding. Mathematical understanding is an integral part in learning mathematics. In order to understand mathematics student should able to modify a representative form to another form; uses diagram models and symbols to represent a concept; define the concept verbally and written; identifying the qualities of a concept and recognising the requirements of a concept; identify and create the

problem; using models, diagrams and symbols to represent concept; and change one form representative to another.<sup>1</sup>

However, in fact, many students face difficulties in understanding mathematics in school teaching. Based on preeliminary studies on geometric transformation subject found that only 28,38 % students able to modify a representative form to another form; 24,13 % students able to use diagram models and symbols to represent a concept; 29,13% students able to define the concept verbally and written; 30% students able to identify the qualities of a concept and recognising the requirements of a concept; 24% students able to identify and create the problem; 26,63% students able to use models, diagrams and symbols to represent concept; and 19,89% students able to change one form representative to another.<sup>2</sup>

The following data showed that most students faced difficulties in understanding mathematics particularly in geometric transformation. There are some reasons why, students face problem in solving geometric transformation problems. One of them is most students did not see geometric transformation visually, they imagine the object learning as abstract thing. As the result, their understanding toward geometric transformation is in low level. There is a need a reforming in teaching geometric transformation in school. Teacher should use a media that accomplish students in understanding geometric transformation virtually.

One of suggestions in reforming mathematics education is the use of concrete and virtual model of mathematics concept<sup>3</sup>. For that reasons, most countries, some schools have applied the integration of teaching and learning instruction help by using technology. Technology is very important for learning and teaching in this era<sup>4</sup>. The use of technology in 21st century is perceived as benefit in supporting teaching and learning for students in secondary level. Many mathematics software can be used to support teaching mathematics such as Geogebra, Geometer' Sketchpad, Math Lab and so on. The using such of mathematical software would have benefit for both teachers and students<sup>5</sup>. Many studies have affirmed that using mathematical software has had contributed and best impact on teaching-learning strategies. The applying software will enhance students performance and maintain attractiveness in class because of visuality by using technology. It has become indispensable to make mathematics teaching processes fun and enjoyable.<sup>6</sup>

The use of technology in pedagogical process is growing due to the increasing a number of using gadget. In learning mathematics, there are many reasons of using technology in teaching and learning, one of them is to visualise and explore insight mathematics with optimum possibility such as in learning geometric transformation<sup>7</sup>. A transformation is a conceptual that should be taught at school in all grades. Furthermore, National Council of Teachers Mathematics state that high school students should understand and represent translations, reflections, rotations, and dilatations of object in a plane by using sketches, coordinates, vectors, function notation and

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<sup>1</sup> National Council of Teacher Mathematics, *Principle and Standard for School Mathematics*. Reston, VA: NCTM, 2000.

<sup>2</sup> Documentation MAN 3 Banda Aceh, Observation Result, Date 11 November 2018.

<sup>3</sup> National Council of Teacher Mathematics, *Principle and Standard for School Mathematics*. Reston, VA: NCTM, 2000

<sup>4</sup> Bekene, T, "Implementation of GeoGebra a Dynamic Mathematical Software for Teaching and Learning of Calculus in Ethiopia". *International Journal of Scientific & Engineering Research*, 11(9), 838–860. 2020, <https://doi.org/10.14299/ijser.2020.09.01>.

<sup>5</sup> Wijaya, T.T., Ying, Z., & Suan, L. "Using Geogebra in Teaching Plane Vector". *Journal of Innovative Mathematics Learning*, 3(1), 15–23, 2020.

<sup>6</sup> Yeliz Celen. "Student Opinions on the Use of Geogebra Software in Mathematics Teaching", *TOJET: The Turkish Online Journal of Educational Technology*, 19(4), 84, 2020.

<sup>7</sup> Khalil, M., Khalil, U., & ul Haq, Z, "Geogebra as a Scaffolding Tool for Exploring Analytic Geometry Structure and Developing Mathematical Thinking of Diverse Achievers". *International Electronic Journal of Mathematics Education*, 14(2), 427–434, 2019, <https://doi.org/10.29333/iejme/5746>.

matrices, and they should use various representations to help understand the effect of simple transformations and their compositions<sup>8</sup>.

The following statement means that learning geometric transformation requires dynamic and efficient teaching media. Dynamic Geometry System such as Geogebra is one of the solutions used in learning geometric transformation. There are many research relate to the use of mathematics software have been conducted globally, however, there are few research have been conducted locally, Aceh-Indonesia<sup>9,10,11</sup>. Some researchers point out that the integration of technology into their educational system depend on the availability of the software, the Geogebra has dynamic properties that allow proofing on geometric transformation<sup>12</sup>, able to visualize transformations by manipulating shapes<sup>13</sup>. For the reasons, there is a need in conducting research relate to the use of Geogebra in teaching geometric transformation.

## METHOD

This research is an explanatory mixed methods design. The subjects of this study were 3 students of XI MAN 3 Banda Aceh. The selection was based on assessment for mathematical understanding test; that given for 23 students, the assessment results were grouped into three categories: the lowest, the average and the highest test score which one student scored the lowest marks, one student score average and one student scored the highest mark.

This study aims to explore students' mathematical understanding abilities when they answer questions of mathematical understanding after learning mathematics using Geogebra software. Data collecting using analysis of student answers in solving mathematical understanding problems. Their written scripts were analysed, and then reserachers interview them in order to get more detailed information about their mathematical understanding.

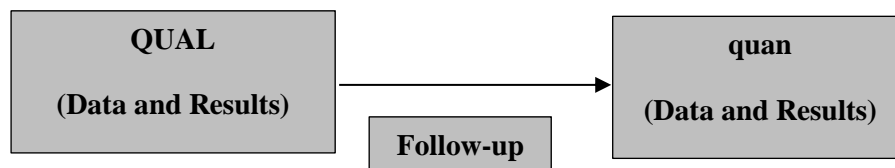


Figure 1. Types of Explanatory Mixed Methods Design<sup>14</sup>

Box = data collection and results

<sup>8</sup> National Council of Teacher Mathematics, *Principle and Standard for School Mathematics*, Reston, VA: NCTM, 2000.

<sup>9</sup> Wijaya, T.T., Jianlan, T., & Purnama, A, "Developing an Interactive Mathematical Learning Media Based on the TPACK Framework Using the Hawgent Dynamic Mathematics Software". *Emerging Technologies in Computing*, 318–328, 2020, <https://doi.org/10.1007/978-3-030-60036-5>.

<sup>10</sup> Wijaya, T.T., Ying, Z., & Suan, L, "Using Geogebra in Teaching Plane Vector". *Journal of Innovative Mathematics Learning*, 3(1), 15–23, 2020.

<sup>11</sup> Wijaya, Tommy Tanu, Tang, J., Li, L., & Purnama, A, "Implementing Dynamic Mathematics Software in Calculus II for Engineering Students: Quadratic Surfaces". *Software Engineering and Algorithms*, 230, 480–491, 2021, [https://doi.org/10.1007/978-3-030-77442-4\\_41](https://doi.org/10.1007/978-3-030-77442-4_41).

<sup>12</sup> Tola Bekene, "Implementation of GeoGebra a Dynamic Mathematical Software for Teaching and Learning of Calculus in Ethiopia", *International Journal of Scientific & Engineering Research*, 11(9), 838, 2020.

<sup>13</sup> Xiangquan Yao and Azita Manouchehri, "Teacher Interventions for Advancing Students' Mathematical Understanding", *Educ. Sci.*, 10, 7; doi:10.3390/educsci10060164, 2020, [www.mdpi.com/journal/education](http://www.mdpi.com/journal/education).

<sup>14</sup> Creswell, John, *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches*. Los Angeles: SAGE, 2017, page. 231.

Uppercase letters = major emphasis  
Lowercase letters = minor emphasis

## RESULTS AND DISCUSSION

As described previously, the test that is arranged to the indicators of understanding concept, namely: 1) Modify a representative form to another form, 2) Uses diagram models and symbols to represent a concept, 3) Define the concept verbally and written, 4) Identifying the qualities of a concept and recognizing the requirements of a concept, 5) Identify and create the problem, 6) Using models, diagrams and symbols to represent concept, 7) Change one form representative to another. Then tests were conducted on 23 students consisting of 11 male students and 12 female students. Furthermore, the results are given a value according to the scoring guidelines for understanding the concept of ability.

The following is the result of a classification of students' conceptual understanding abilities of class XI MAN 3 Banda Aceh.

**Table 1**  
Students' Conceptual Understanding Scoring of Class XI MAN 3 Banda Aceh.

No	Student's initials	Scoring Based on the Concept Understanding Ability Scoring Rubric					Total	Achievement Percentage of Understanding Concept	The Category
		No.1	No.2	No.3	No.4	No.5			
<b>1</b>	<b>SD</b>	<b>2</b>	<b>1</b>	<b>1</b>	<b>3</b>	<b>3</b>	<b>10</b>	<b>50%</b>	<b>Average</b>
2	AU	2	1	1	1	2	7	35%	Average
3	AK	2	1	1	1	1	6	30%	Lowest
4	AW	2	3	1	3	3	12	60%	Average
5	AS	2	1	1	2	2	8	40%	Average
6	AZ	2	1	2	3	3	11	55%	Average
7	DA	3	2	3	3	3	14	70%	Highest
8	DP	2	3	3	3	3	14	70%	Highest
9	FI	2	1	1	3	3	10	50%	Average
10	FJ	2	2	3	3	3	13	65%	Average
12	FK	2	1	3	3	3	12	60%	Average
12	GR	2	1	1	0	2	6	30%	Lowest
<b>13</b>	<b>RD</b>	<b>2</b>	<b>1</b>	<b>1</b>	<b>2</b>	<b>0</b>	<b>6</b>	<b>30%</b>	<b>Lowest</b>
14	HI	2	4	1	3	3	13	65%	Average
15	IK	4	2	1	3	3	13	65%	Average
16	JA	2	4	1	3	3	13	65%	Average
17	MA	2	1	1	2	3	9	45%	Average
18	MD	2	1	1	3	3	10	50%	Average
<b>19</b>	<b>ST</b>	<b>2</b>	<b>4</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>15</b>	<b>75%</b>	<b>Highest</b>
20	NA	2	2	2	3	3	12	60%	Average
21	NI	2	2	2	4	4	14	70%	Highest
22	RA	2	2	4	3	3	14	70%	Highest
23	RS	2	1	1	3	3	10	50%	Average

Based on the test result in Table 1, subject selection is based on certain student ability criteria, namely 1 student with highest, 1 student with average, and 1 student with lowest. The data collection technique used was a concept understanding test sheet and interview guidelines.

To conduct data on the results of the students' conceptual understanding ability test, a scoring guide is needed as a reference for marking scores to each question. The scoring guidelines criteria used are as follows:

**Table 2**  
 Rubric Scoring Test

Skor	Description
0	Does not answer the question
1	Less precise in understanding concepts and incorrect answer
2	Less precise in understanding concepts but the answer is correct
3	Can understand concepts well, but there are errors in the calculation process and incorrect answers
4	Can understand concepts well, calculation process and correct answer

The categories of students' conceptual understanding abilities are as follows

**Table 3**  
 Criteria for the Level of Understanding of Mathematical Concepts

Percentage	Level of Understanding
$0\% < P \leq 33,3\%$	Lowest
$33,3\% < P \leq 66,6\%$	Average
$66,6\% < P \leq 100\%$	Highest

**1. Point of View for Implementing DGS.**

The teacher on transformation geometry subject explores students mathematical comprehending by using Dynamic Geometry System (DGS). The teacher gradually started to introduce several step how to operate DGS with the attention that student's capability to involve in using software DGS as well as the capability to follow up in teaching and learning process.

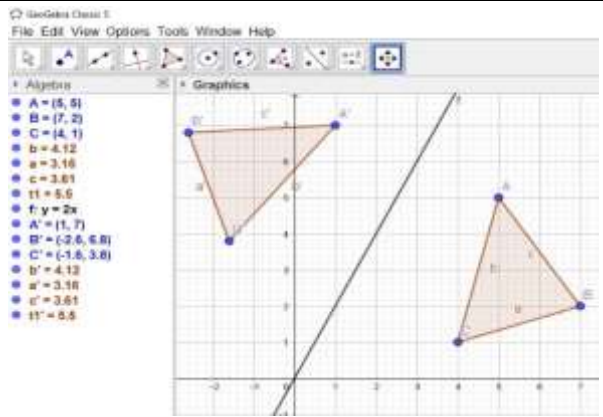
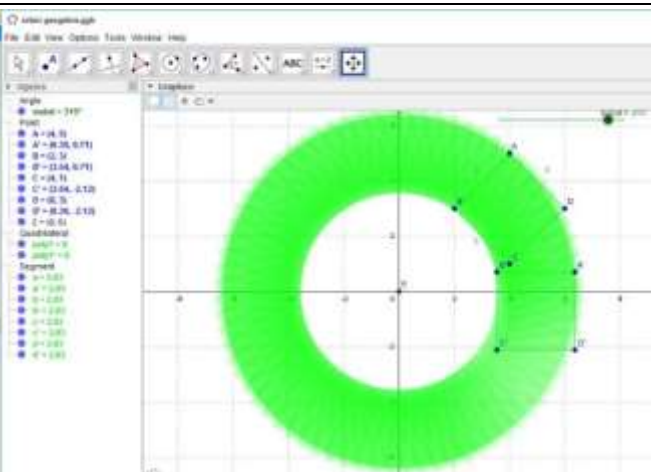
DGS is a computer program that can be applied in helping at transformation geometry understanding, especially in geometry and algebra. It affirms that Isaac's opinion that DGS is a computer program to assist teach mathematics and assessing students' learning, particularly geometry and algebra<sup>15</sup>. The program is delightful to utilize and even well-known because it has been applied by millions of people around the world, by students, teachers, as well as lectures.

The researcher utilized the thought of DGS to develop student understanding that forwards with the DGS mathematical software. Moreover, it describes tutorial DGS in geometry transformation spot as follow:

<sup>15</sup> Isaac Benning, "Enacting Core Practices of Effective Mathematics Pedagogy with GeoGebra", *MERGA*, 23(2), 102, 2021.

**Table 4**

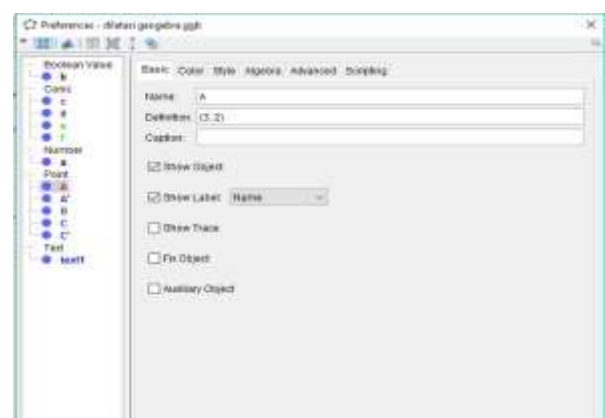
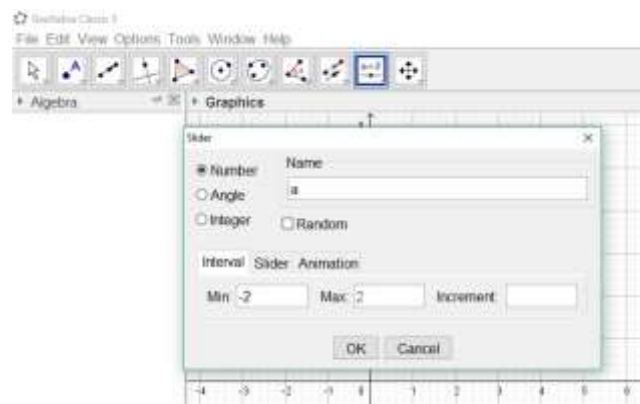
Step of teacher to Figure out applying DGS software in Geometry Transformation

Steps for reflecting in DGS	The Image
<p><b>A line</b></p> <ul style="list-style-type: none"> <li>• Select New Point then pick out point A(5,5) and right-click on that point, click Rename to name the points. Do the same for point B(7,2) and point C(4,1).</li> <li>• Select the Polygon toolbar to create a triangle ABC and connect the points A, B, and C.</li> <li>• Type <math>y=2x</math> in the input then press enter, then right-click on the line click Rename and change it to h.</li> <li>• Select Reflect Object in Line to reflect triangle ABC, click point A and line <math>h:y=2x</math>. Likewise, the points B and C process is the same as point A.</li> <li>• After being reflected, make a triangle A''B''C''. Select the Polygon toolbar and connect the points A'' B'' and C'' it will show the results of the reflection of triangle ABC on the line h in the form of triangle A''B''C''</li> </ul>	 <p style="text-align: center;">Figure 2. Reflecting a line</p>  <p style="text-align: center;">Figure 3. a rotation</p>
<p><b>A Rotation</b></p> <ol style="list-style-type: none"> <li>1. We open up Geogebra, then create a polygon. Click Polygon, select the four points in the Graphics View, click return to the starting point.</li> <li>2. Then make a point at E(0,0). Click the toolbar Point and click the point (0,0).</li> <li>3. the next step, create a slider. Click Slider and click anywhere in the Graphics View, then a dialog box appears. Select Angel change the name from <math>\alpha</math> to aangle then click apply.</li> </ol>	

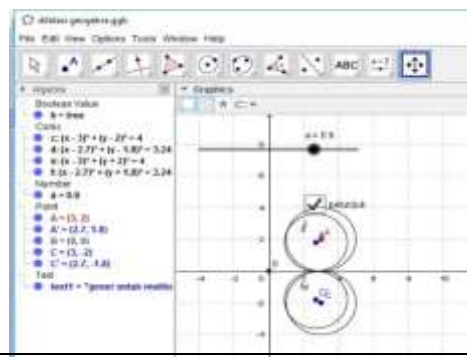
4. The polygon has the name poly1. Next we type input "Rotate[poly1,angle,E]" then enter.
5. To make it look more attractive, you can change the color of the shadow on the side of the algebra or polygon. select Right-click on the image, pick Object properties, select Color, and then specify the color that you want.
6. Then we see the shift of the trace by right-clicking on the shadow and we turn on the trace on it, then we turn on the animation on the corner slider, Animation on, so we get the result as shown in the image.

#### ***A Dilatation***

7. First, we create a point A at coordinates (3,2) and a point B at coordinates (0,0). Then create a slider by clicking the tool slider. Then fill in the following form.
8. Input the following format to display a circle centered on point A that we created earlier.  $c = \text{Circle}[A,2]$ . Create one more point, namely point C (3,-2) and a circle centered at point C by inputting the format like this:  $e = \text{Circle}[C,2]$ .
9. create a point as a result of the dilatation of point A by inputting the following format:  $A' = \text{Dilate}[A,a,B]$ . And create one more point as a result of point C dilatation by inputting the following format:  $C' = \text{Dilate}[C,a,B]$ .
10. Input the following format to display a circle centered on k A"  $d = \text{Dilate}[\text{Circle}[A,2],a,B]$  Input the following format to display a circle centered at point C".  $f = \text{Dilate}[\text{Circle}[C,2],a,B]$ .
11. Moreover, we can see the movement clearly, then right-click on each circle select Object properties, in Basic, check the Show Trace section.



12. To write text, we can make it by clicking Insert Text



13. Create Checkbox. First, type what is the contents of the instructions, then click the Check Box to Show/Hide Objects icon, in the Caption column type instructions, then select the column for the object which is next to click which is all the contents of the instructions themselves that were previously created from Insert Text .



14. Change the color of one of the circles. Next, select the slider, there is a reduction and enlargement with results like the picture below

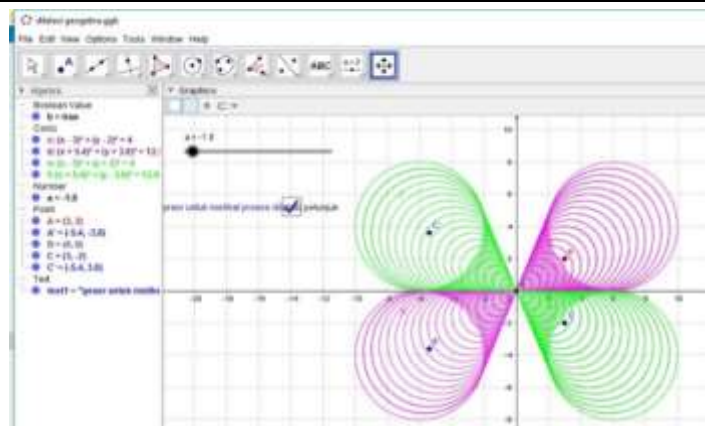


Figure 3.A Dilatation

Source: Software Geogebra classic 5

After the teacher gave the tutorial, it was continued by giving a test. An essay consists of 5 questions, each of which has its level of difficulty based on indicators of students' mathematical understanding of the DGS software-assisted transformation material. Based on data from 25 students, it was revealed, that 3 students were selected based on tests to analyze student answers sheet based on 3 of students' mathematical understanding levels, namely high, medium and low categories.

## 2. Basic Result

In addition, the teacher's understanding of the Geogebra software is the main point to the successful applying it. Xian and azita said that Teacher interventions that encourage students'



developth toward growing a understanding of a mathematical idea<sup>16</sup>. Moreover, the teacher must indeed explain the transformation geometry first accompanied by the worksheet then clarified using the Geogebra software. At this stage, they also have the opportunity to prove between the theories was learned by practicing directly on Geogebra software. ÖzlemÇeziktürk represents that student answers describe that Geogebra is a good tool to study<sup>17</sup>, the same result said by Yeliz; creates mathematics teaching class enjoy and fascinating in doing Geogebra software<sup>18</sup>. which aims to reduce verbalism and material abstraction and that can increase students' understanding of the material that is implemented.

Next, the written sheet was analyzed in order to clarify the student’s mathematical understanding in using GDS software. The analyzed will interpret a specific level of students' mathematical understanding exploring.

**Problem 1**

Given some triangles and their points as follow

Triangle’s Name	Object point	Object point	Object point
ABC	A(2,2)	B(6,2)	C(6,5)
DEF	D(-2,2)	E(-2,5)	F(-6,2)
GHI	G(2,-2)	H(2,-5)	I(6,-5)
JKL	J(-6,-5)	K(-6,-5)	L(-2,-5)
MNO	M(1,4)	N(1,4)	O(0,-3)

Sketch one of the triangles then the distance of the vertices to O (0,0) is doubled and ½.

Table 5  
Coding Framework Students’ Mathematical Understanding

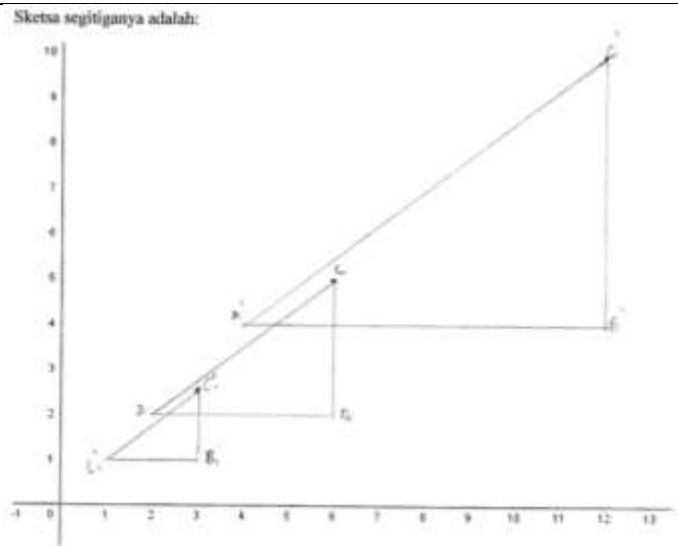
Level of Understanding	The student’s answer sheet														
1. Modify a representative form to another form	<table border="1"> <thead> <tr> <th rowspan="2">Titik objek</th> <th colspan="2">Titik Bayangan</th> </tr> <tr> <th>Faktor skala ke 2</th> <th>Faktor skala ke ½</th> </tr> </thead> <tbody> <tr> <td>A. ( 2 , 2 )</td> <td><math>\begin{pmatrix} x' \\ y' \end{pmatrix} = 2 \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}</math></td> <td><math>\begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}</math></td> </tr> <tr> <td>B. ( 6 , 2 )</td> <td><math>\begin{pmatrix} x' \\ y' \end{pmatrix} = 2 \begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 12 \\ 4 \end{pmatrix}</math></td> <td><math>\begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}</math></td> </tr> <tr> <td>C. ( 6 , 5 )</td> <td><math>\begin{pmatrix} x' \\ y' \end{pmatrix} = 2 \begin{pmatrix} 6 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 12 \\ 10 \end{pmatrix}</math></td> <td><math>\begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2.5 \end{pmatrix}</math></td> </tr> </tbody> </table>	Titik objek	Titik Bayangan		Faktor skala ke 2	Faktor skala ke ½	A. ( 2 , 2 )	$\begin{pmatrix} x' \\ y' \end{pmatrix} = 2 \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	B. ( 6 , 2 )	$\begin{pmatrix} x' \\ y' \end{pmatrix} = 2 \begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 12 \\ 4 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$	C. ( 6 , 5 )	$\begin{pmatrix} x' \\ y' \end{pmatrix} = 2 \begin{pmatrix} 6 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 12 \\ 10 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2.5 \end{pmatrix}$
Titik objek	Titik Bayangan														
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C. ( 6 , 5 )	$\begin{pmatrix} x' \\ y' \end{pmatrix} = 2 \begin{pmatrix} 6 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 12 \\ 10 \end{pmatrix}$	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2.5 \end{pmatrix}$													

<sup>16</sup> Xiangquan Yao and Azita Manouchehri, “Teacher Interventions for Advancing Students’ Mathematical Understanding”, *Educ. Sci.* 2020, 10, 1., 164; doi:10.3390/educsci10060164, 2020, www.Mdpi.com/journal/education.

<sup>17</sup> Özlem Çeziktürk, “Understanding functional dependency on Dynamic Geometry Systems (DGS): Napoleon and von Aubel Theorems on Geogebra”, *International Technology and Education Journal*, 4(1), 20, 2020, http://itejournal.com.

<sup>18</sup> Yeliz Celen, “Student Opinions on the Use of Geogebra Software in Mathematics Teaching”, *TOJET: The Turkish Online Journal of Educational Technology*, 19(4), 88, 2020.

2. Uses diagram models and symbols to represent a concept



**Interview Analyses**

- Researcher : “Are there obstacle in determining the shadow of a particular object?”
- Student (SD) : “No, just put it into the matrix, so we can imagine the dilatation”
- Researcher : “is there an obstacle to the issue?”
- Student (SD) : “difficulty in drawing”
- Researcher : “what is a difficult step, that a shadow point has been obtained?”
- Student (SD) : “I’m not precise in locating coordinates of the object point”

Base on data, interview results, and student test analyses, then it can describe the student’s understanding ST (the highest), SD (the average), and RD (the lowest) can easily solve problems, there is no issue in determining the shadow spot from their objects. But at the stage of modeling the diagram to represent the concept, the students are still confused to define a shadow of double or a half, for SD and RD the most essential thing is revealed an understanding of the student's early concept is also confused in determining the coordinates of the shadow object point.

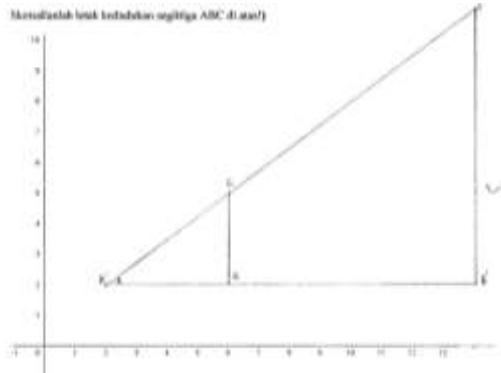
Finally, the criteria were changing a representative form to another for each student category are already fulfilled, however for indicators using a diagram model and symbol to represent the concept, only ST was qualified that can change a representative object, and RD is unfulfilled. It is concluded that all students easily comprehend the algebra concept, which is determined a shadow by using a matrix but when reverting to the diagram form they face difficulty answering.

**Problem 2**

Triangle’s Name	Object point	Object point	Object point	Centre point	Scale factor
ABC	A(2,2)	B(6,2)	C(6,5)	P(2,2)	K=3
DEF	D(-2,2)	E(-2,5)	F(-6,2)	P(1,1)	
GHI	G(2,-2)	H(2,-5)	I(6,-5)	P(-1,-1)	
JKL	J(-6,-5)	K(-6,-5)	L(-2,-5)	P(2,-2)	
MNO	M(1,4)	N(1,4)	O(0,-3)	P(2,-1)	

Select one of the above triangle to designate its shadow point with factor scale  $K = 3$ .

**Table 6**  
Coding Framework Students' Mathematical Understanding

Level of Understanding	The student's answer sheet									
3. Define the concept verbally and written	<table border="1"> <thead> <tr> <th rowspan="2">Titik objek</th> <th>Titik Bayangan</th> </tr> <tr> <th>Faktor skala ke 3</th> </tr> </thead> <tbody> <tr> <td>A.. ( 2, 2 )</td> <td><math>\begin{pmatrix} x' \\ y' \end{pmatrix} = 3 \begin{pmatrix} 2-1 \\ 2-1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}</math></td> </tr> <tr> <td>B.. ( 6, 2 )</td> <td><math>\begin{pmatrix} x' \\ y' \end{pmatrix} = 3 \begin{pmatrix} 6-1 \\ 2-1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 14 \\ 2 \end{pmatrix}</math></td> </tr> <tr> <td>C.. ( 6, 5 )</td> <td><math>\begin{pmatrix} x' \\ y' \end{pmatrix} = 3 \begin{pmatrix} 6-1 \\ 5-1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 14 \\ 11 \end{pmatrix}</math></td> </tr> </tbody> </table>  <p>Membuatlah titik bayangan segitiga ABC di atas!</p> <p>Proses diatas dinamakan <b>DILATASI</b> atau perbesaran dengan notasi D. Skala atau faktor perbesaran dinotasikan dengan k dan k tidak harus bilangan bulat.</p> <p>Dari permasalahan diatas dapat disimpulkan:</p> <p><u>Definisi Dilatasi adalah suatu transformasi mengubah ukuran (luas/persebaran atau panjang) menuju bentuk acuan yang sama tidak mengubah bentuk bangun tersebut.</u></p> <p>Matrik yang digunakan untuk mencari transformasi Dilatasi adalah <math>\begin{bmatrix} x' \\ y' \end{bmatrix} = k \begin{bmatrix} x-p \\ y-q \end{bmatrix} + \begin{bmatrix} p \\ q \end{bmatrix}</math></p> <p>Sifat dari Dilatasi adalah (a) jika <math>k &gt; 1</math> maka dilatasinya diperbesar (b) jika <math>k = 1</math> maka tidak mengalami perubahan ukuran dan tetap (c) jika <math>0 &lt; k &lt; 1</math> maka dilatasinya diperkecil (d) jika <math>-1 &lt; k &lt; 0</math> maka dilatasinya direflesi (e) jika <math>k = -1</math> tidak mengalami perubahan bentuk dan ukuran dan tidak terbalik arah (kadang fungsi dilatasi (f) jika <math>k &lt; -1</math> maka dilatasinya diperbesar dan terbalik arah (g) dilatasi pada kehidupan sehari-hari adalah perbesaran objek pada mikroskop dan kaca pembesar, foto yang dicetak, diperbesar atau diperkecil.</p>	Titik objek	Titik Bayangan	Faktor skala ke 3	A.. ( 2, 2 )	$\begin{pmatrix} x' \\ y' \end{pmatrix} = 3 \begin{pmatrix} 2-1 \\ 2-1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$	B.. ( 6, 2 )	$\begin{pmatrix} x' \\ y' \end{pmatrix} = 3 \begin{pmatrix} 6-1 \\ 2-1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 14 \\ 2 \end{pmatrix}$	C.. ( 6, 5 )	$\begin{pmatrix} x' \\ y' \end{pmatrix} = 3 \begin{pmatrix} 6-1 \\ 5-1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 14 \\ 11 \end{pmatrix}$
Titik objek			Titik Bayangan							
		Faktor skala ke 3								
A.. ( 2, 2 )		$\begin{pmatrix} x' \\ y' \end{pmatrix} = 3 \begin{pmatrix} 2-1 \\ 2-1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$								
B.. ( 6, 2 )		$\begin{pmatrix} x' \\ y' \end{pmatrix} = 3 \begin{pmatrix} 6-1 \\ 2-1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 14 \\ 2 \end{pmatrix}$								
C.. ( 6, 5 )		$\begin{pmatrix} x' \\ y' \end{pmatrix} = 3 \begin{pmatrix} 6-1 \\ 5-1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 14 \\ 11 \end{pmatrix}$								
4. Identifying the qualities of a concept and recognizing the requirements of a concept										
5. Identify and create the problem										
6. Using models, diagrams and symbols to represent concept										
7. Change one form representative to another										

Based on test and interview data 3 students, from indicators (3), (4), and (5), both ST, SD and RD could correctly understand the concepts of definitions and the nature of dilatation. Visible from students' answers can write the result of understanding the theory of dilatation. But this doesn't line up with (7) for sd and rd still confused about representing a shadow point on courtesies coordinates, and the scale of the factor. The elementary school revealed how to draw a shadow point if what  $k = 3$  wanted, was confused in determining the starting point. This is because it is

still weak early concepts in modifying absis and coordinates when these are the basic skill for drawing.

## DISCUSSION

The result of discussion becomes a quantifier when using their Geogebra software can easily represent picture only by selecting features in an application, but in reality, when required manually drawing, the student faced hard a representative to visualize the image. Thus, it needs to enhance the concept when using software, Geogebra is used to help students represent shadow objects. On the contrary, it is not necessarily the absolute reference to the concept.

Teachers need to reinforce verbal concepts and written, and then prove with the use of Geogebra software. Then, teachers need to review and corroborate student representation of the reflecting concept. The difficult representation in modeling into the image is also an impediment to research. Xiangquan & Azita<sup>19</sup>, students facing an obstacle in modifying an object. Moreover, enhancing image reviewing consists of teacher interventions that external interventions might be an essential action to attract learners in reviewing and reflecting taken on mathematical images and the product of modified object reflecting. Strengthen by Ngoc The students make more progress when applying GeoGebra to teaching than traditional teaching methods.<sup>20</sup> Students need to resolve within these categories more than other categories while enhancing mathematical understanding of transformations.

## CONCLUSION

Based on analysis from students' assessment tests on transformation subject and interviews as well, then researchers may conclude that: from seven criteria mathematical understanding of transformation subject is already five categories were qualified by ST, SD and RD, which is already a model that students' understanding of developing geometry using Geogebra software. However, it is an important indicator that has not yet changed a representative form to another. Only ST (the lowest) students can represent the images, while SD and RD still have difficulties. Because they understood merely the concept of algebra to manipulate the symbol into a matrix, but when modifying the image, they were having complexity, which was an obstacle to the determining of objects that were shadows in the courtesies coordinates.

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<sup>19</sup> Xiangquan Yao and Azita Manouchehri, "Teacher Interventions for Advancing Students' Mathematical Understanding", *Educ. Sci.* 2020, 10, 164; doi:10.3390/educsci10060164, 2020, www. Mdpi.com/journal/education.

<sup>20</sup> Ngoc-Giang Nguyen, "The discovery teaching of the problem of finding the shortest distance with the help of Geogebra software in Vietnam", *Educational Research and Reviews*, 16(8), pp. 343-356, 2021, DOI: 10.5897/ERR2021.4182

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